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Design of Insensitive Multirate Aircraft Control Using Optimized Eigenstructure Assignment

Y. Patel,* R. J. Patton,† and S. P. Burrows‡
University of York, York, England Y01 5DD, United Kingdom

This paper presents a new method for the design of constant-gain controllers for digital aircraft systems monitored and updated at multiple frequencies. It addresses two main problems associated with such a scheme: the adverse effects of intersample cross coupling and the high-amplitude control signals that normally result from a multirate feedback structure. An examination of the problem indicates that the two objectives present conflicting design criteria. Thus, a solution must seek to achieve an acceptable compromise between the minimization of control effort and the assignment of a prescribed modal structure. An unconstrained numerical optimization procedure, based on eigenstructure assignment using full-state feedback, has been developed to provide for this compromise. A suitable choice of weighting factors, introduced into the scalar cost function defined for the optimization, allows for the computation of two alternative control laws. They differ only in the emphasis placed on the two conflicting design objectives. The lateral motion of a light aircraft is the test system employed to illustrate the application of the method. The controllers are evaluated using a fully nonlinear simulation of the aircraft. The results fully demonstrate the eigeninsensitivity of the multirate feedback solution and the flexibility offered by the eigenstructure-assignment optimization approach in both cases.

Introduction

MULTIRATE control has been investigated by many in the past, motivated by the need to optimize the performance of sampled systems with varying dynamic modes and to make efficient use of available hardware resources. The trend has been recently accelerated with the emergence of multiprocessor architectures. The multirate philosophy is particularly suited to applications in the aerospace industry; increasing demands on airborne computing have created serious data throughput constraints on flight control processors. In addition to this, the disparate bandwidths of various subsystems within the aircraft demand a wide range of update rates. Combined, these requirements have led to multirate, multiprocessor computation of flight information.[1,2] However, such advances in airborne computing have not been accompanied by the development of any one comprehensive control design methodology capable of full exploitation of multirate system capabilities.

The ability of multirate systems to enhance closed-loop system robustness by virtue of the design freedom offered by their periodic operation has been recognized by several researchers.[3-5] However, very few multirate control methods that directly incorporate these robustness properties as an achievable design objective and that furthermore, address the practical problems associated with the implementation of multirate controllers in an interactive multi-input, multi-output (MIMO) environment exist. This paper describes a class of periodic systems: multirate structures with multiple-input/fixed-output (MIFO) sample rates (see Fig. 1), and presents a control design technique based on eigenstructure assignment that makes full use of the multirate system design freedom to prescribe closed-loop system robustness. The robustness quality obtained using this design technique is the insensitivity of the closed-loop eigenstructure to variations in the nominal system dynamics that may arise from inaccurate identification/modeling or disturbances.

The two main limitations of MIFO multirate control systems are that they are highly sensitive to intersample crossfeed effects and typically demand oscillatory, high-amplitude control effort. These problems are particularly characteristic of sampling schemes that have fast control input updates within a slower output and state signal monitoring time frame. Clearly, the structure is highly prone to undesired effects propagating through the system during the slower output period and causing an adverse response before they can be monitored and corrected. Therefore, the primary objective of an effective multivariable, multirate design method is to minimize this potentially destabilizing crossfeed effect while maintaining control activity within acceptable actuation limits.

Eigenstructure assignment is used to design a state feedback gain matrix such that the resulting closed-loop system has a desired set of eigenvalues and right eigenvectors. Necessary and sufficient conditions for such a gain matrix were determined by Moore,[6] who characterized the freedom, beyond pole placement, available in MIMO systems to influence closed-loop behavior by eigenvector specification. The eigenvalues characterize closed-loop modes in terms of frequency and damping while the right eigenvectors indicate the amount that each mode contributes to the transient response of every system output. The eigenstructure is a particularly important design consideration for the MIFO multirate feedback problem. A design incorporating a reduction in modal interaction will minimize crossfeed effects and thus provide good closed-loop disturbance rejection properties.

Previous work by the authors[7-10] has demonstrated that MIFO multirate control structures can exploit the design capa-

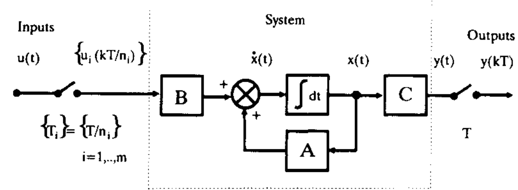

Fig. 1 Multirate system with MIFO sample rates.

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*Research Assistant, Department of Electronics.

†Senior Lecturer, Department of Electronics. Senior Member AIAA.

‡Research Student, Department of Electronics.

bilities of the method more fully than corresponding single rate and continuous-time systems. This arises from the ability of MIFO systems to generate extra design freedom for the specification of eigenvectors by increasing the input sample rates. The extra freedom can be used to assign desired modal structures perfectly, at the cost of very oscillatory state and control behavior.[7] For practically implementable feedback solutions, a more effective use of the extra design freedom is to assign near-perfect modal structures with simultaneous control effort minimization.[8]

The design approach considered in this paper provides an optimized solution to the MIFO eigenstructure-assignment problem. The optimization task is to minimize a scalar cost function that measures both the demanded control effort and the modal interactions of the closed-loop system.[11,12] Weighting factors control the contribution of each of these properties to the overall cost function. In this way, the feedback control design can be tailored to address the specific needs of the MIFO sampled system.

An example that considers the design of two multirate, state feedback control laws for the lateral subsystem of an aircraft (linearized at a given operating point) demonstrates the application of this method. The control laws differ in the emphasis placed on the two design objectives thus illustrating the tradeoff between the reduction of control effort and the decoupling of modes.

MIFO Multirate System Equations

The amount of information necessary for the MIFO system pole assignment problem requires equations that represent state and output vectors at main periodic instants and a control matrix that describes input effects only at the instants that they change. This results in a concise, low-dimensional system description derived as follows. Consider the linear time-invariant description of a controllable and observable continuous-time system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$, and $y \in \mathcal{R}^p$ are the state, control, and output matrices, respectively. If the continuous system is subjected to m inputs that change from one constant value to another at m successive, uniformly spaced time instants $t = kT_i$ where

$$T_i = T/n_i, \qquad i = 1,\ldots,m \tag{2}$$

then the transition of states over time period T can be classed as periodic. Interval T is the least common multiple of all input sample periods (see Fig. 1) referred to as the *main interval of sampling*. If the sample period ratios are rational, then

$$T_i/T_j = n_j/n_i, \qquad i,j = 1,\ldots,m \tag{3}$$

(where n_i, n_j have no common factor) and there exists a lowest common multiple n_0 and a lowest common sample period T such that

$$p_i = n_0/n_i, \qquad i = 1,\ldots,m$$

$$T_i = p_i T_b \tag{4a}$$

$$n = n_1 + n_2 + \cdots + n_m \tag{4b}$$

The ratio $T_b = T/n_0$ defines the base sample period. These sampling interrelationships describe a MIFO sampled system; they are used to derive the following set of transition equations that codify the invariant nature of the system over the interval T:

$$x[(k+1)T] = \Phi[n_0, 0] \times (kT) + \Psi(kT) \tag{5}$$

$$\Psi(kT) = \sum_{i=1}^{m} \sum_{j=0}^{n_i - 1} \Phi[n_0, q]\Gamma_i[q, jp_i]u_i(kT + jT_i) \tag{6a}$$

$$q = (j+1)p_i \tag{6b}$$

Signal u_i is the ith control input and the periodic transition matrix is defined as

$$\Phi[i,j] = \begin{cases} [\exp(AT_b)]^{i-j} & i > j \\ I_n & i = j \\ 0 & i < j \end{cases} \tag{7}$$

and

$$\Gamma_i[(k+1), k] = \int_{kT_b}^{(k+1)} \Phi[(k+1), \tau]B_i(\tau)\,d\tau \tag{8}$$

B_i denotes the ith column of the original system control matrix of Eq. (1). The MIFO sampled system transition equations have outputs sampled at instants $t = kT$, while the inputs are updated at different rates related by the sampling relationships of Eqs. (3) and (4). The pole placement problem relates to the assignment of n poles of the system description given by Eqs. (5) and (6).

Eigenstructure-Assignment Preliminaries

For the system of Eq. (1), the eigenvalue-assignment problem is to design a constant feedback gain matrix K such that the state feedback control

$$u(t) = Kx(t), \qquad K \in R^{m \times n} \tag{9}$$

produces a closed-loop system matrix $(A + BK)$ that has a set of a priori poles $\{\lambda_1\ \lambda_2 \cdots \lambda_n\}$, $\lambda_i \in \mathcal{C}$. Assuming (A, B) are completely controllable and $1 < m < n$, a range of feedback matrices K will provide the solution.[6] Eigenstructure assignment uses the extra degrees of freedom in the underdetermined solution to specify the closed-loop eigenvectors, $V = \{\nu_1\ \nu_2 \cdots \nu_n\}$, $\nu_i \in \mathcal{C}$ corresponding to the desired self-conjugate set Λ. For an insensitive closed-loop solution, the eigenvectors are chosen either to be as mutually orthogonal as possible or to have a specific modal structure. The latter is particularly useful for the design of flight control systems as the aircraft modal structure relates to its flight handling qualities[13] and can be specified to enhance aircraft maneuverability. The use of eigenstucture assignment for aerospace control applications has been examined by many[11,12] due to this ability to incorporate aircraft handling qualities design criteria in a direct manner. Also, the insensitivity properties thus obtained ensure that a control law (designed for a linear system description at a nominal operating point) will maintain desired closed-loop performance for small perturbations in the nominal operating condition, such as those experienced in a nonlinear environment.

A measure of the insensitivity of a solution at a given operating point is the conditioning of the right eigenvectors to variations in the elements of the closed-loop system matrices (due to uncertainty[13]), as given by

$$\kappa(V) = \|V\|_2 \|V^{-1}\|_2 \tag{10}$$

This provides an upper bound on the sensitivities of the individual poles as measured by the condition number

$$c_i = \|\nu_i\|_2 \|\ell_i\|_2 / |\ell_i^T \nu_i| \geq 1 \tag{11}$$

where $\{\ell_i\}$ denote the set of left eigenvectors associated with $\{\nu_i\}$. Assume $\{\lambda_i\} \in \mathbb{R}$. The eigenstructure-assignment problem is to select a feedback matrix K and a set of linearly independent right eigenvectors V such that $\kappa(V)$ is minimized and which satisfies

$$\nu_i = (\lambda_i I - A)^{-1}\beta\omega_i, \qquad i = 1, \ldots, n \tag{12}$$

where

$$\omega_i = KN_{\lambda i}k_i$$

is the design parameter. $N_{\lambda i} \in \mathbb{R}^{n\times m}$ can be defined by considering the transformation

$$V\Lambda V^{-1} = A + BK, \qquad \Lambda = \{\lambda_1 \lambda_2 \cdots \lambda_n\} \tag{13}$$

A rearrangement of Eq. (13) gives

$$[(\lambda_i I - A) : -B]R_{\lambda i}k_i = 0 \tag{14}$$

where $R_{\lambda i} = [N_{\lambda i}^T M_{\lambda i}^T]^T$ defines the matrix whose columns span the solution space. For a given $R_{\lambda i}$, vector $k_i \in \mathbb{R}^{m\times 1}$ is selected such that $N_{\lambda i}k_i$ is equivalent to (or approximates) the desired eigenvector ν_i. The feedback matrix K is determined as

$$K = [M_{\lambda 1}k_1 \, M_{\lambda 2}k_2 \cdots M_{\lambda n}k_n]V^{-1} \tag{15}$$

For the single-rate eigenproblem of Eq. (11), an achievable set of eigenvectors is limited by the nullity of the solution space $N_{\lambda i}$. In general, $m < n$ thus indicating that an overdetermined problem must be solved for which a maximum of m elements of the desired eigenvectors can be achieved exactly. For the MIFO multirate system, the dimension of $R_{\lambda i}$ can be extended beyond that produced by a fixed single-rate system, by virtue of the periodic description used for the MIFO sampling scheme.

Systematic Technique for Control Law Design Compromise

For the eigenproblem of Eq. (11), the two design objectives of interest for the multirate feedback control problem are in competition; a gain matrix of low norm will not, in general, assign a desirable set of eigenvectors that reduce modal interaction and vice versa. This conflict can be addressed by formulating the eigenproblem as an optimization task; the degree to which each design objective is satisfied being determined by weighting factors in an overall scalar cost function. Minimizing this cost function using numerical optimization will produce a control law with the desired balance of each closed-loop multirate system quality. The overall scalar cost function is defined as

$$J = J_N + J_M \tag{16}$$

where J_N is a measure of the size of elements in the full state gain matrix F, given by

$$J_N = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_{ij}(f_{ij})^2 \tag{17}$$

f_{ij} is the (i, j)th element of F and α_{ij} are nonnegative weighting factors. J_M is a measure of the proximity of achievable right eigenvectors $\{\nu_i\}$ to the desired set $\{\nu_{id}\}$ defined as

$$J_M = \frac{1}{2}\sum_{i=1}^{n}\beta_i[\bar{\nu}_i - \bar{\nu}_{id}]^T[\nu_i - \nu_{id}] \tag{18}$$

Scalars β_i are, again, nonnegative weighting factors and an overbar denotes the complex-conjugate vector. The contribution of each design objective to the optimization criteria is controlled by weighting elements α_{ij} and β_i. The precise effect of the choice of α_{ij} and β_i on the eigenproblem solution is dependent on the system under examination and thus cannot be generalized. (The work of Burrows[9] demonstrates clearly the tradeoff between accurate assignment of the desired modal structure and minimization of feedback gain elements achieved by varying weighting elements α_{ij} and β_i for a particular system.) Provided J is small, control laws that satisfy adequately both reduced modal interaction and low control effort are produced. The cost function J is minimized using analytically derived gradients along with the Davidon-Fletcher-Powell optimization technique.[14] Details of this design procedure are well documented in the publications of Burrows[9] and Burrows and Patton[10] and hence not included here. The sensitivity of control designs thus produced can be monitored by the conditioning measures of Eqs. (10) and (11), while control effort magnitude can be assessed by the gain matrix norm $\|\cdot\|_2$.

Example

This example demonstrates the use of the multirate pole placement technique proposed in this paper. The design of a stability augmentation system for the lateral motion of a light aircraft is illustrated. The aircraft has a high level of interaction of the natural modes in the unstable, open-loop state. Feedback control is used to stabilize the aircraft. A generic modal structure for the desired closed-loop system exists, providing a realistic control and performance tradeoff task for the multirate feedback problem. The activity of aircraft actuators is also an important consideration; control signals should ideally be within specified limits. The design of a suitable multirate feedback control scheme incorporating both requirements is clearly a challenging task.

Two control laws are formulated; both attempt to minimize the modal interactions of the closed-loop system in addition to reducing the norm of the gain matrix. They differ in the emphasis placed on each; the first places an emphasis on the norm minimization of the gain matrix (by selecting the α_{ij} elements to be larger than β_i elements; e.g., an order of magnitude greater than β_i), whereas the second is designed to primarily achieve the desired modal structure (selecting larger β_i elements). A selection of α_{ij} and β_i of similar magnitude would allocate equal design attention to the optimization procedure. The lateral dynamics of the light aircraft linearized for a stick-fixed configuration at an airspeed of 33 ms^{-1} are described by the system matrices[12,15]:

$$A = \begin{bmatrix} -0.2770 & 0 & -32.9000 & 9.8100 & 0 \\ -0.1033 & -8.5250 & 3.7500 & 0 & 0 \\ 0.3649 & 0.0000 & -0.6390 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{19a}$$

$$B = \begin{bmatrix} -5.4320 & 0.0000 \\ 0.0000 & -26.6400 \\ -9.4900 & 0.0000 \\ 0.0000 & 0.0000 \\ 0.0000 & 0.0000 \end{bmatrix} \tag{19b}$$

where the state vector is

$$\begin{bmatrix} v \\ p \\ r \\ \varphi \\ \psi \end{bmatrix} = \begin{bmatrix} \text{sideslip} \\ \text{roll rate} \\ \text{yaw rate} \\ \text{bank angle} \\ \text{yaw angle} \end{bmatrix} \tag{20}$$

The input vector, comprising the demanded lateral motion control surface deflections, is

$$\begin{bmatrix} \tau \\ \xi \end{bmatrix} = \begin{bmatrix} \text{rudder angle} \\ \text{aileron angle} \end{bmatrix} \tag{21}$$

To satisfy handling qualities criteria, an appropriate conjugate set of the desired closed-loop poles for a stability augmentation system is

$$\lambda_1 = -4 \qquad \text{(Roll mode)} \tag{22a}$$

$$\lambda_{2,3} = -1.5 \pm j\,1.5 \qquad \text{(Dutch roll mode)} \tag{22b}$$

$$\lambda_4 = -1 \qquad \text{(Heading angle mode)} \tag{22c}$$

$$\lambda_5 = -0.05 \qquad \text{(Spiral mode)} \tag{22d}$$

The preferred eigenvector structure relating these modes to the flight handling properties of the aircraft is well documented.[12] The Dutch roll mode should appear predominantly in the sideslip and yaw states, while bank angle and roll rate coupling is suppressed. The roll mode should be decoupled from yaw rate and sideslip and the spiral mode is required not to appear in sideslip velocity to avoid sideslip in steady turns. A desirable choice of eigenvectors would therefore be

$$\nu_{1d} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ * \\ * \end{bmatrix}, \qquad \nu_{2d} = \begin{bmatrix} 1 \\ 0 \\ * \\ 0 \\ * \end{bmatrix} \pm j \begin{bmatrix} * \\ 0 \\ 1 \\ 0 \\ * \end{bmatrix}$$

$$\nu_{3d} = \bar{\nu}_{2d}, \qquad \nu_{4d} = \begin{bmatrix} * \\ * \\ * \\ * \\ * \end{bmatrix}, \qquad \nu_{5d} = \begin{bmatrix} 0 \\ * \\ * \\ 1 \\ * \end{bmatrix} \tag{23}$$

where the asterisk is used to denote that no particular preference is required. The multirate model of Eq. (5) is formed first. For the controllability condition[7,8] to be satisfied, the rudder τ and aileron ξ sampling frequencies must be related by

$$T_\tau = T/3, \qquad T_\xi = T/2 \tag{24}$$

The main interval of sampling is selected to be $T = 0.1$ s. This does not render any of the modes uncontrollable and hence all of the desired eigenvalues of Eq. (22) may be assigned. Using the optimization procedure, two control laws are designed: the first places an emphasis on reducing the norm of the gain matrix, and the second on achieving the desired modal structure. The gain matrices are denoted K_{NORM} and K_{MODAL}, respectively, and are given below:

Table 1 Performance measures of multirate feedback designs K_{NORM} and K_{NORM}

Performance measures	K_{MODAL}	K_{NORM}
Pole sensitivities (c_i)		
$\lambda_1 = -4$	2.1074	23.3446
$\lambda_{2,3} = -1.5 \pm j1.5$	12.1080	36.8392
$\lambda_4 = -1$	6.0523	38.3804
$\lambda_5 = -0.05$	1.1273	1.9870
$\kappa(V)$	29.3396	139.7424
$\lVert K \rVert_2$	0.4383	0.1508

The right eigenvectors assigned by K_{NORM} are given by

$$\nu_1 = \begin{bmatrix} 1.0000 \\ 0.8602 \\ 0.0354 \\ -0.2097 \\ -0.0106 \end{bmatrix}, \qquad \nu_2 = \begin{bmatrix} 1.0000 \\ 0.3562 \\ -0.0194 \\ -0.1627 \\ -0.0165 \end{bmatrix} \pm j \begin{bmatrix} 0.0000 \\ -0.1447 \\ -0.0639 \\ -0.0675 \\ 0.0269 \end{bmatrix}$$

$$\nu_3 = \bar{\nu}_2, \qquad \nu_4 = \begin{bmatrix} 1.0000 \\ 0.3524 \\ -0.0881 \\ -0.3384 \\ -0.0834 \end{bmatrix}, \qquad \nu_5 = \begin{bmatrix} -0.0027 \\ 0.0083 \\ -0.0498 \\ -0.1659 \\ 1.0000 \end{bmatrix} \tag{26}$$

Those assigned by K_{MODAL} are

$$\nu_1 = \begin{bmatrix} -0.0002 \\ 1.0000 \\ 0.0000 \\ -0.2267 \\ 0.0170 \end{bmatrix}, \qquad \nu_2 = \begin{bmatrix} 1.0000 \\ -0.0004 \\ 0.0336 \\ 0.0005 \\ -0.0254 \end{bmatrix} \pm j \begin{bmatrix} 0.0000 \\ 0.0011 \\ -0.0437 \\ -0.0003 \\ 0.0040 \end{bmatrix}$$

$$\nu_3 = \bar{\nu}_2, \qquad \nu_4 = \begin{bmatrix} 1.0000 \\ -0.2858 \\ -0.0761 \\ -0.2600 \\ -0.0933 \end{bmatrix}, \qquad \nu_5 = \begin{bmatrix} 0.0015 \\ 0.0084 \\ -0.0501 \\ -0.1662 \\ 1.0000 \end{bmatrix} \tag{27}$$

The overall conditioning, pole sensitivity measures, and $\lVert\cdot\rVert_2$ figures produced by gain matrices K_{NORM} and K_{MODAL} are shown in Table 1.

The tabulated results show that control matrix K_{MODAL} has superior insensitivity properties to the feedback control K_{NORM}. Comparing the right eigenvectors associated with each control design [Eqs. (26) and (27)] with the desired set [Eq. (23)], K_{MODAL} is seen to achieve the required decoupling far more closely than K_{NORM}. This is to be expected from the emphasis of the K_{MODAL} design on accurate modal assignment. Matrix

$$K_{\text{NORM}} = \begin{bmatrix} -0.0064 & 0.0226 & -0.0078 & -0.0485 & -0.0041 \\ 0.0051 & 0.0294 & -0.0065 & -0.0500 & -0.0080 \\ 0.0187 & 0.0366 & -0.0041 & -0.0501 & -0.0052 \\ -0.0636 & -0.0236 & 0.0089 & 0.0413 & -0.0016 \\ -0.0834 & -0.0285 & 0.0093 & 0.0484 & -0.0005 \end{bmatrix} \tag{25a}$$

$$K_{\text{MODAL}} = \begin{bmatrix} 0.0373 & 0.2553 & 0.1615 & -0.0443 & 0.0017 \\ 0.0285 & 0.0046 & 0.1437 & 0.0059 & 0.0113 \\ 0.0190 & -0.2508 & 0.1240 & 0.0567 & 0.0208 \\ -0.0032 & 0.0098 & 0.1356 & 0.1483 & 0.0224 \\ -0.0020 & -0.1582 & 0.1158 & 0.2113 & 0.0332 \end{bmatrix} \tag{25b}$$

K_{NORM}, however, has a far lower norm figure than K_{MODAL}.

To demonstrate the performance of both gain matrices, the results of implementing the controllers in a fully nonlinear simulation of the aircraft for an initial perturbation of 0.02 rad in heading angle and 0.2 rad in bank angle are shown in Figs. 2 and 3. (Note that the control input responses shown in Figs. 2 and 3 are the rudder and aileron actuator outputs that drive the lateral subsystem.) The responses demonstrate clearly the difference in emphasis placed on the two control laws. Control law K_{MODAL} produces modal interactions which enhance the performance of the aircraft lateral subsystem and correspondingly alleviates the overall control task required to achieve the desired closed-loop responses. This serves to reduce the control effort demanded by the aircraft system far more effectively than a direct minimization of the gain matrix norm, as for control law K_{NORM}.

The effect that each control law has on the modal structure of the aircraft can be assessed by the degree of yaw, roll, and sideslip motions induced by the initial perturbations in heading and bank angle. Handling qualities require that the roll mode be decoupled from both yaw and sideslip velocity to provide, for example, well-coordinated aircraft turning abilities. Figures 2 and 3 show that feedback design K_{MODAL} achieves this decoupling with far more success than design K_{NORM}.

It is well known that the initial heading-angle perturbation of Fig. 2 should induce the Dutch roll mode. This occurs from the trailing wing generating extra lift causing the aircraft to roll and sideslip. Weathercock correction from the tail then produces identical motion in the opposite yaw direction. Combined, these motions constitute the Dutch roll mode. Figure 2 shows that the K_{MODAL} design induces significantly less Dutch roll than the K_{NORM} design. This is apparent from the smooth recovery of sideslip, roll, and yaw motion from the initial heading-angle perturbation produced by controller K_{MODAL}. In contrast, roll and sideslip motions of the K_{NORM} controlled system react quite significantly to the initial perturbation. For this maneuver, the accommodating aircraft modal structure of feedback design K_{MODAL} also effects a significant saving in aileron control activity. It demands less than 2% of the control effort required by the inferior design, K_{NORM}.

For the closed-loop system of controller K_{NORM}, the initial roll-angle perturbation of Fig. 3 causes further banking before proper corrective action in the opposite direction is applied. This, in turn, induces sideslip and yawing in the direction corresponding to the roll motion. Such deviations from the desired steady-state trajectory are not exhibited by the control design of K_{MODAL}, which forces the aircraft to its prescribed course smoothly and with minimal sideslip and yaw effects. This improved modal structure is achieved at the cost of slightly increased switched activity of both rudder and aileron control surfaces. Despite this, the total control effort required by controller K_{MODAL} is much less than that demanded by controller K_{NORM}.

The performance of controllers K_{NORM} and K_{MODAL} in correcting the heading- and bank-angle perturbations demonstrate the improved decoupling (and hence improved insensitivity properties) of the latter design. The responses of Figs. 2 and 3 confirm the insensitivity results of Table 1 and demonstrate the effectiveness of the optimized eigenstructure-assignment method in fulfilling the conflicting design objectives of the multirate pole placement problem. Furthermore, the responses show clearly the impact of the closed-loop modal structure on aircraft performance during coordinated maneuvers.

Nonlinear simulations of the aircraft with a +50% variation in the nominal airspeed and for larger initial perturbations in the heading angle show that controllers K_{NORM} and K_{MODAL} maintain a similar standard of desired closed-loop performance. Thus, the controllers can be applied to an extended operational envelope.

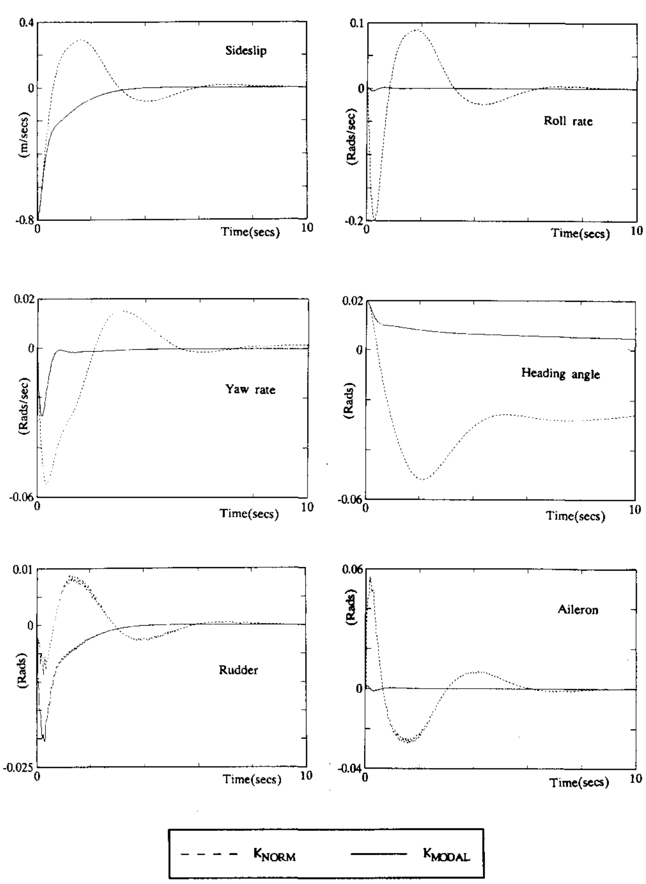

Fig. 2 Responses of K_{NORM} and K_{MODAL} aircraft lateral subsystems to heading-angle perturbation.

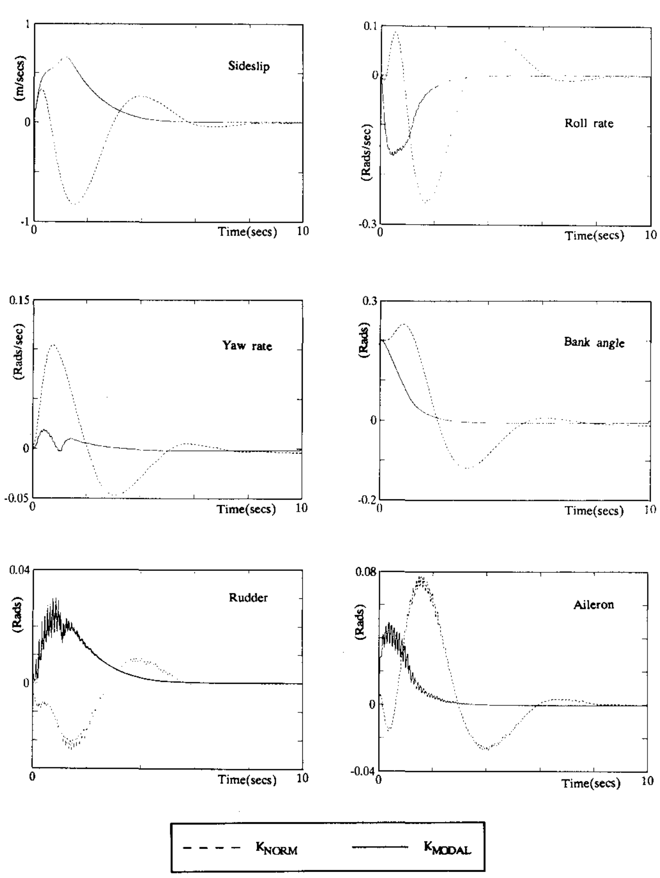

Fig. 3 Responses of K_{NORM} and K_{MODAL} aircraft lateral subsystems to roll-angle perturbation.

Conclusions

The eigenstructure-assignment technique proposed in this paper has been shown to provide solutions that alleviate the two main problems associated with a multirate input, fixed-rate output sampled flight system, namely high-control input effort and adverse intersample effects. The insensitivity that is achieved with modal decoupling ensures that the closed-loop aircraft maintains its prescribed behavior when subjected to uncertainties. This is a necessary quality of a flight control system that is usually designed for a nominal open-loop description generated from aerodynamic analysis of aircraft performance over a given operating envelope. An effective control design must be able to withstand deviations of the nominal system model from the true aircraft behavior due to either modeling inaccuracies or changes in flight conditions. Multirate control schemes offer computational efficiency for automatic flight systems but are, by the very nature of their operation, very sensitive to such uncertainties and disturbance effects. This paper has illustrated the design of two feedback control laws for an aircraft lateral subsystem to demonstrate the capability of multirate sampled structures to provide insensitive solutions for aircraft control problems. Both solutions address the preceding problems just described but differ in the emphasis that is placed on the minimization of each individual objective. The design perspective considered includes the specification of the degree of tradeoff between the two system performance characteristics through the use of the weighting elements associated with each quality. This has been shown to provide an accurate means of compromise between control input minimization and the assignment of a particular set of closed-loop eigenvectors. The performance of the two control laws has been demonstrated using the nonlinear simulation study.

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